EE 434
Lecture 24

Bipolar Small Signal Device Models

Quiz 16

What is a “binning model” and what is the purpose of using “binning models”?

And the number is ….

1 8 7 5 3

6 9 4 2

And the number is ….

1 8 7 5 3

6 9 4 2

4

Quiz 16

What is a “binning model” and what is the purpose of using “binning models”?

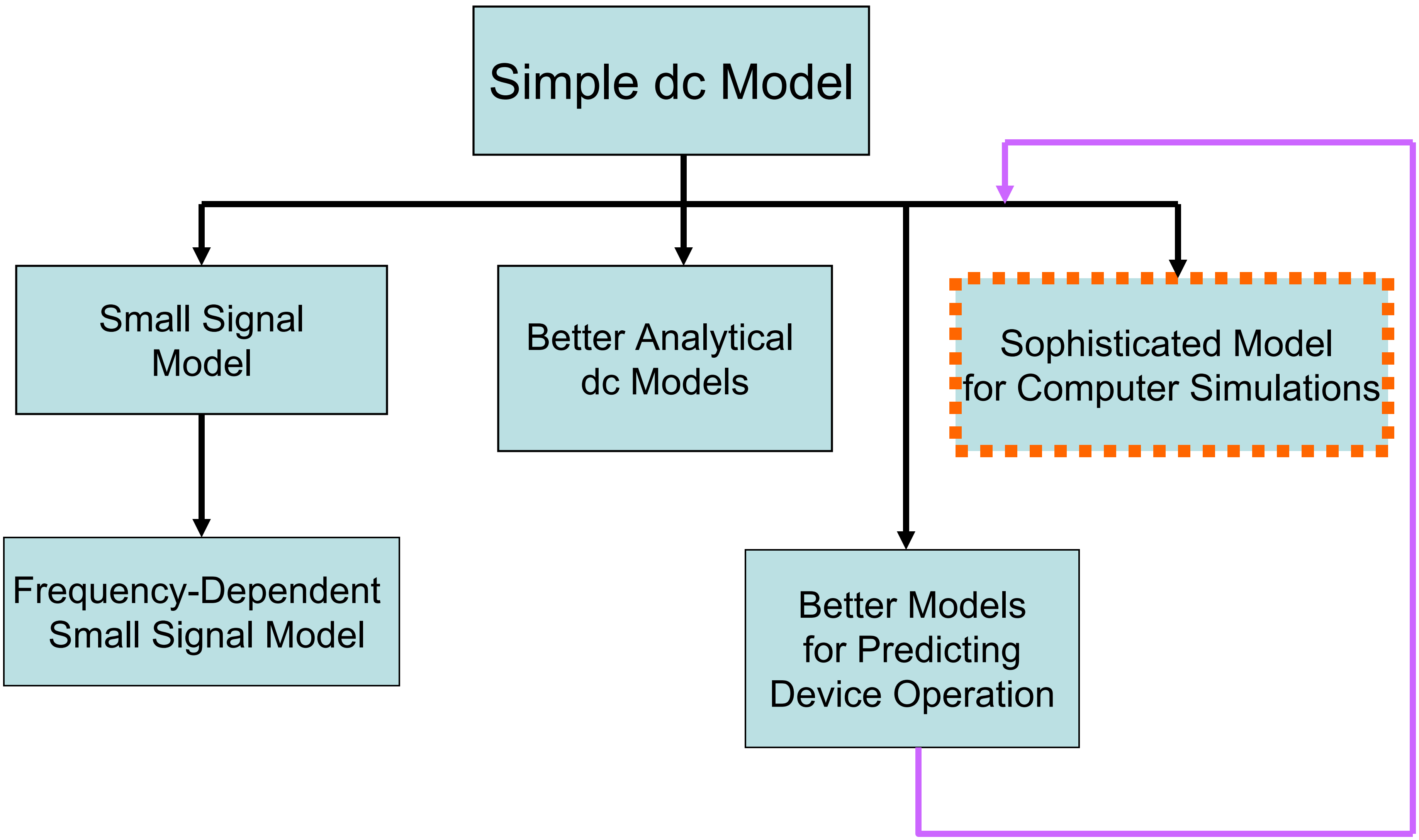
Solution:

A binning model is actually a set of models whereby the model derived for dimensions close to those of a specific device is used rather than using the same model for each device (the functional form of most binning models does not change, simply the parameters in the model)

A good binning model will more closely predict the actual characteristics of a device than a model that does not change with device dimensions.

Review from Last Time

Models for Computer Simulation



Review from Last Time

Concept in modeling is to partition model into two parts, one that characterizes the technology and the other that characterizes the geometric aspects of a device

Technology part of the model common to all devices in a process (Level 1, BSIM4 , PSP models – over 100 paramaters in BSIM 4 model)

Geometric information unique to each device
$\{W, L, N_{RD}, N_{RS}, A_D, A_S, P_D, P_S\}$, (default values used in not specified)

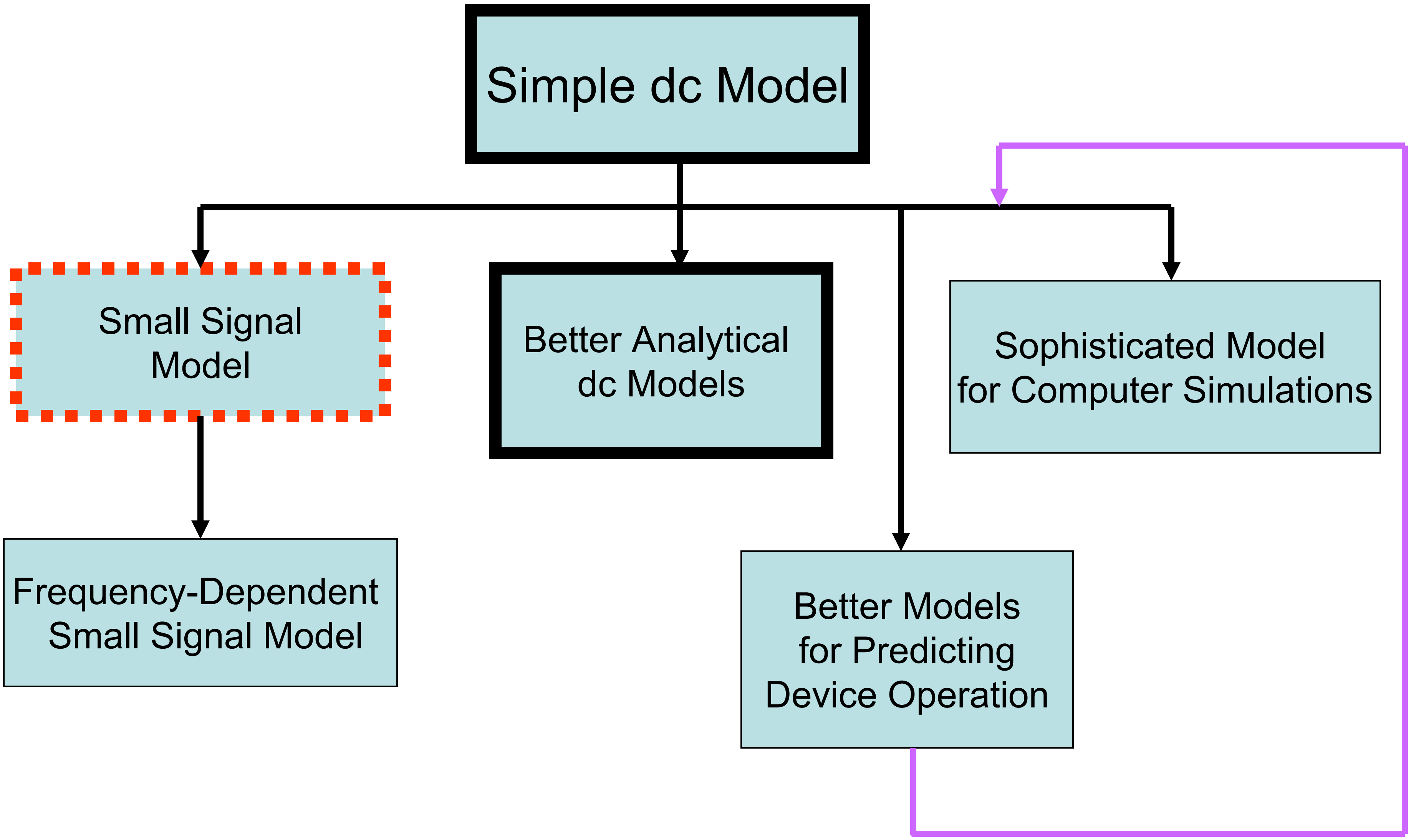
Models based upon physical principles but emperically modified to either simplfy model or improve validity

Geometric description may not be unique

Anticipated parasitics often included at schematic level for design prior to layout

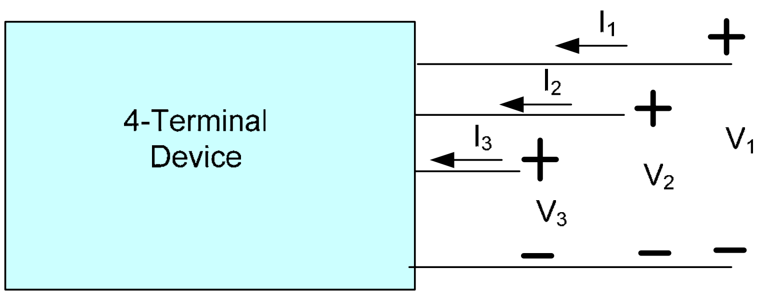
Hierarchy used in models

Bipolar Models



Recall:

Small-Signal Model



$$\left.\begin{aligned} i_1 &= g_1(v_1, v_2, v_3) \\ i_2 &= g_2(v_1, v_2, v_3) \\ i_1 &= g_3(v_1, v_2, v_3) \end{aligned}\right\}$$

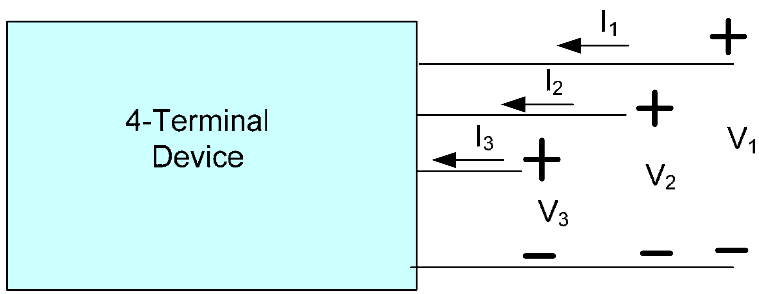
$$i_1 = y_{11} v_1 + y_{12} v_2 + y_{13} v_3$$
$$i_2 = y_{21} v_1 + y_{22} v_2 + y_{23} v_3$$
$$i_3 = y_{31} v_1 + y_{32} v_2 + y_{33} v_3$$

$$\mathbf{y}_{ij} = \left. \frac{\partial \mathbf{f}_i(\mathbf{V}_1, \mathbf{V}_2, \mathbf{V}_3)}{\partial \mathbf{V}_j} \right|_{\vec{V} = \vec{V}_Q}$$

- Small signal circuit model is linear (and unique at a Q-point)
- Small signal equivalent circuits are not unique

Recall:

Small-Signal Model



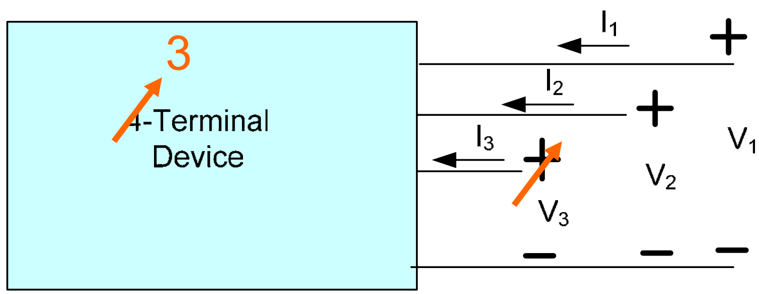
$$\left.\begin{array}{l} i_1 = g_1(v_1, v_2, v_3) \\ i_2 = g_2(v_1, v_2, v_3) \\ i_1 = g_3(v_1, v_2, v_3) \end{array}\right\}$$

Small signal model is that which represents the relationship between the small signal voltages and the small signal currents

For small signals, this relationship should be linear

Recall:

Small-Signal Model



$$
\left.
\begin{aligned}
i_1 &= g_1(v_1, v_2, v_3) \\
i_2 &= g_2(v_1, v_2, v_3) \\
i_1 &= g_3(v_1, v_2, v_3)
\end{aligned}
\right\}
$$

$$
\begin{aligned}
i_1 &= y_{11}v_1 + y_{12}v_2 + y_{13}v_3 \\
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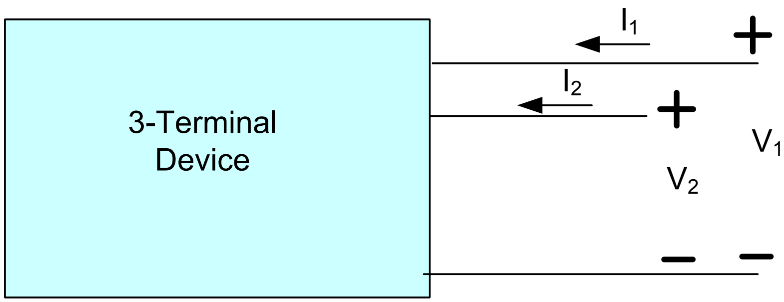
\end{aligned}
$$

$$
y_{ij} = \left. \frac{\partial f_i(V_1, V_2, V_3)}{\partial V_j} \right|_{\vec{V} = \vec{V}_Q}
$$

- Small signal circuit model is linear (and unique at a Q-point)
- Small signal equivalent circuits are not unique

Recall:

Small-Signal Model



$$\left.\begin{array}{l} I_1 = f_1(V_1, V_2) \\ I_2 = f_2(V_1, V_2) \end{array}\right\}$$

Define

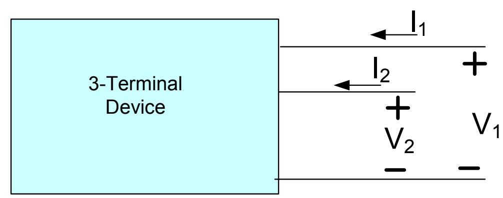
$$i_1 = I_1 - I_{1Q} \qquad v_1 = V_1 - V_{1Q}$$

$$i_2 = I_2 - I_{2Q} \qquad v_2 = V_2 - V_{2Q}$$

Small signal model is that which represents the relationship between the small signal voltages and the small signal currents

Recall:

Small-Signal Model

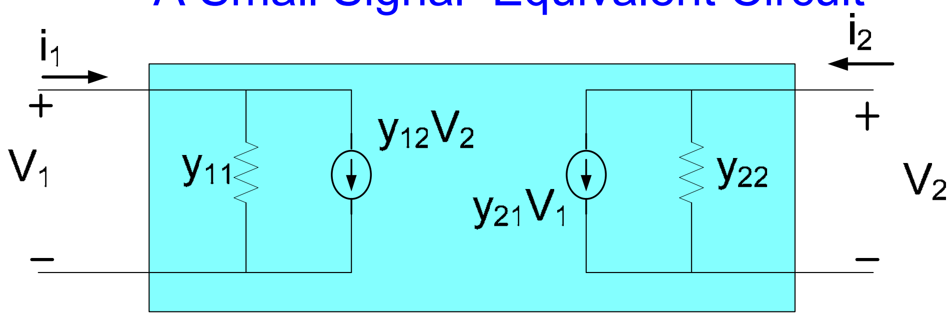


$$i_1 = y_{11}v_1 + y_{12}v_2$$
$$i_2 = y_{21}v_1 + y_{22}v_2$$

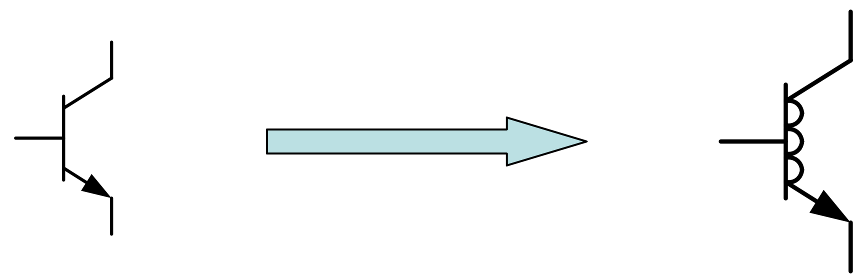
$$y_{ij} = \left.\frac{\partial f_i(V_1,V_2)}{\partial V_j}\right|_{\vec{V}=\vec{V}_Q}$$

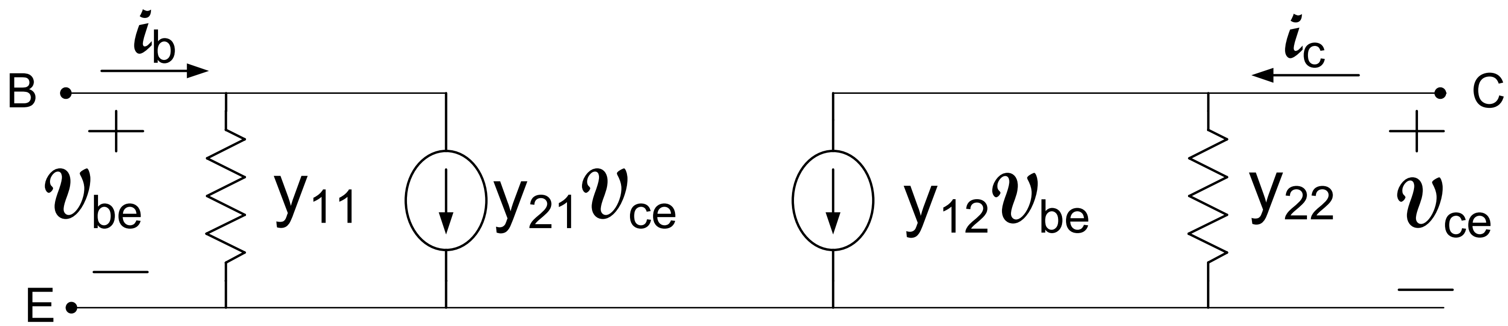
$$\vec{V} = \begin{pmatrix} V_{1Q} \\ V_{2Q} \end{pmatrix}$$

A Small Signal Equivalent Circuit



Small Signal BJT Model





$$y_{11} = \left.\frac{\partial I_B}{\partial V_{BE}}\right|_{Q-PT} \overset{defn}{=} g_\pi \qquad y_{21} = \left.\frac{\partial I_C}{\partial V_{BE}}\right|_{Q-PT} \overset{defn}{=} g_m$$

$$y_{12} = \left.\frac{\partial I_B}{\partial V_{CE}}\right|_{Q-PT} \overset{defn}{=} \; ? \qquad y_{22} = \left.\frac{\partial I_C}{\partial V_{CE}}\right|_{Q-PT} \overset{defn}{=} g_o$$

Small Signal BJT Model

$$y_{11} = \left.\frac{\partial I_B}{\partial V_{BE}}\right|_{Q-PT} \overset{defn}{=} g_\pi \qquad y_{21} = \left.\frac{\partial I_C}{\partial V_{BE}}\right|_{Q-PT} \overset{defn}{=} g_m$$

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Region of Operation for Small Signal Model : Forward Active

"1" $$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

"2" $$I_C = \beta I_B \left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

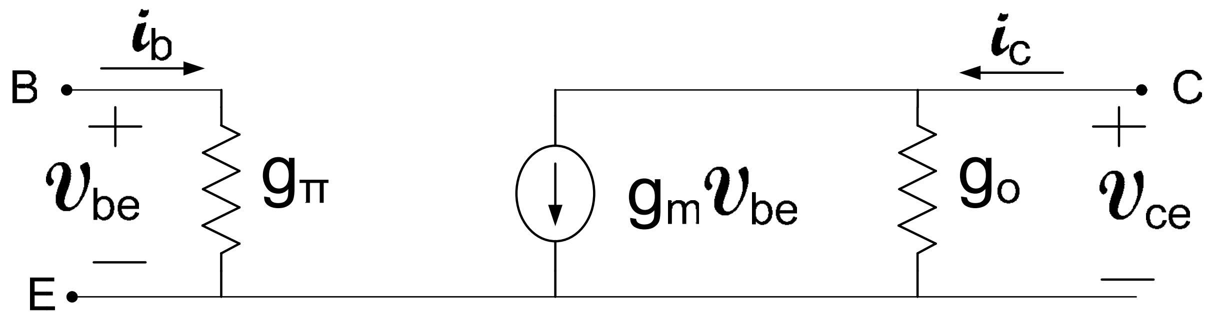
$$y_{11} = \left.\frac{\partial I_B}{\partial V_{BE}}\right|_{Q-PT} = \frac{1}{V_t}\left.\left(\frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}\right)\right|_{Q=PT} = \frac{I_{BQ}}{V_t} = \frac{I_{CQ}}{\beta V_t}$$

$$y_{12} = \left.\frac{\partial I_B}{\partial V_{CE}}\right|_{Q-PT} = 0$$

$$y_{21} = \left.\frac{\partial I_C}{\partial V_{BE}}\right|_{Q-PT} = \frac{1}{V_t}\left.\left(J_S A_E e^{\frac{V_{BE}}{V_t}}\right)\right|_{Q-PT} = \frac{I_{CQ}}{V_t}$$

$$y_{22} = \left.\frac{\partial I_C}{\partial V_{CE}}\right|_{Q-PT} = \frac{1}{V_{AF}}\left[J_S A_E e^{\frac{V_{BE}}{V_t}}\Big|_{Q-PT}\right]_o \cong \frac{I_{CQ}}{V_{AF}}$$

Small Signal BJT Model

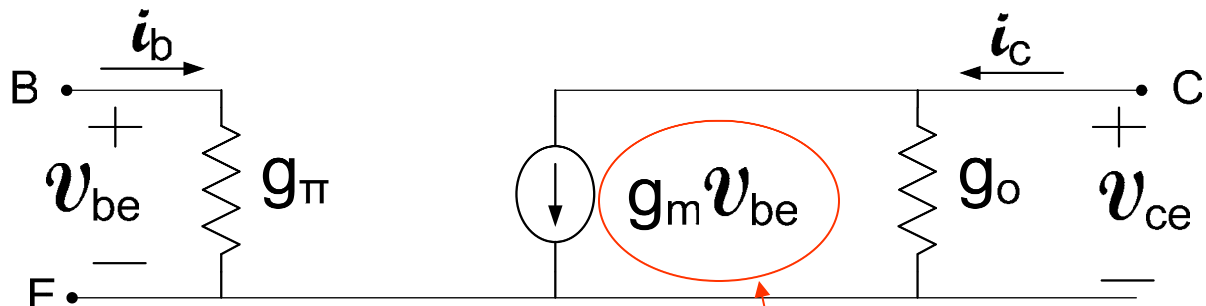


$$g_m = \frac{I_{CQ}}{V_t}$$

$$g_\pi = \frac{I_{CQ}}{\beta V_t}$$

$$g_o \cong \frac{I_{CQ}}{V_{AF}}$$

Small Signal BJT Model



$$g_m = \frac{I_{CQ}}{V_t} \qquad g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_o \cong \frac{I_{CQ}}{V_{AF}}$$

Observe :

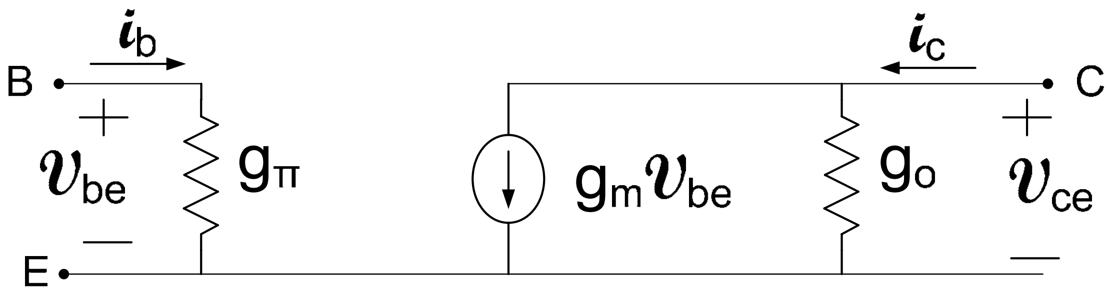
$$g_\pi v_{be} = i_b$$

$$g_m v_{be} = i_b \frac{g_m}{g_\pi}$$

$$\frac{g_m}{g_\pi} = \frac{\left[\frac{I_Q}{V_t}\right]}{\left[\frac{I_Q}{\beta V_t}\right]} = \beta$$

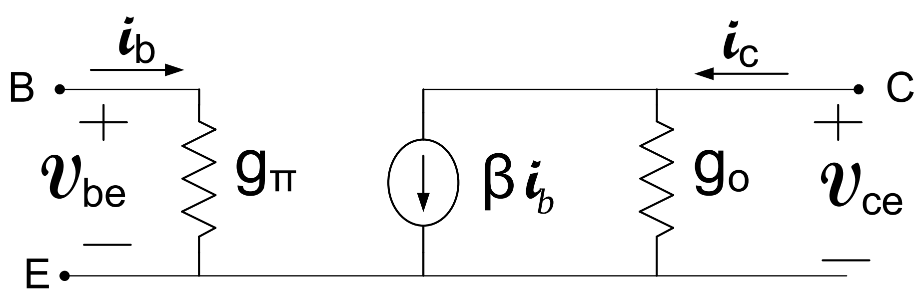
$$g_m v_{be} = \beta i_b$$

Small Signal BJT Model



$$g_m = \frac{I_{CQ}}{V_t} \qquad g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_o \cong \frac{I_{CQ}}{V_{AF}}$$

Alternate equivalent small signal model

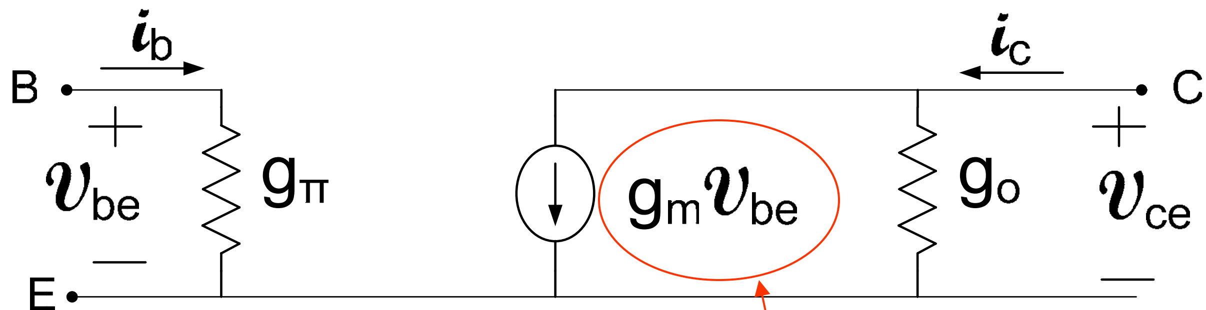


$$g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_o \cong \frac{I_{CQ}}{V_{AF}}$$

Properties of the BJT

- Alternate Equivalent Small Signal Model
- Relative magnitude of small signal parameters
  - Simplified small signal model

Small Signal BJT Model



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Observe :

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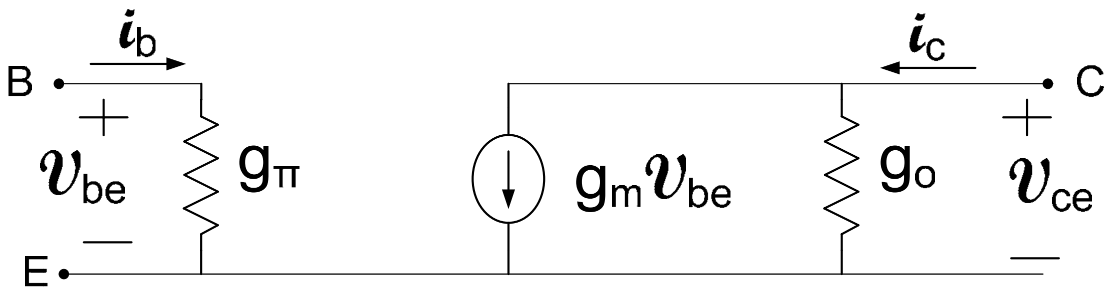
$$g_m v_{be} = i_b \frac{g_m}{g_\pi}$$

$$\frac{g_m}{g_\pi} = \frac{\left[\frac{I_Q}{V_t}\right]}{\left[\frac{I_Q}{\beta V_t}\right]} = \beta$$

$$g_m v_{be} = \beta i_b$$

Small Signal BJT Model



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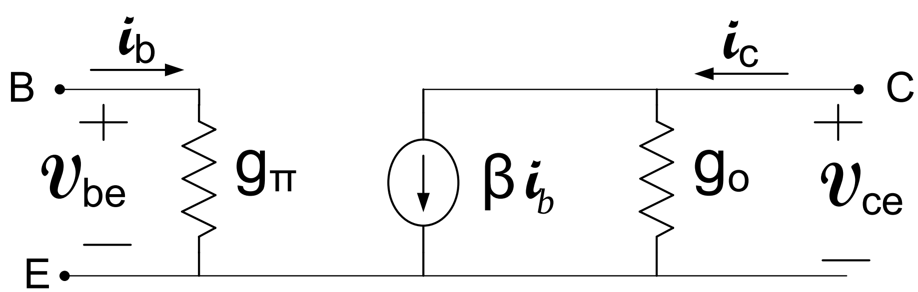
Alternate equivalent small signal model



$$g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_o \cong \frac{I_{CQ}}{V_{AF}}$$

Properties of the BJT

- Alternate Equivalent Small Signal Model

Magnitude of small signal parameters

Relative Magnitude of Small Signal Parameters

$$g_m = \frac{I_{CQ}}{V_t} \qquad g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_o \cong \frac{I_{CQ}}{V_{AF}}$$

$$\frac{g_m}{g_\pi} = \frac{\left[\dfrac{I_Q}{V_t}\right]}{\left[\dfrac{I_Q}{\beta V_t}\right]}$$

$$\frac{g_\pi}{g_o} = \frac{\left[\dfrac{I_Q}{\beta V_t}\right]}{\left[\dfrac{I_Q}{V_{AF}}\right]}$$

$$g_m >> g_\pi >> g_o$$

Often the go term can be neglected in the small signal model because it is so small

Relative Magnitude of Small Signal Parameters

$$g_m = \frac{I_{CQ}}{V_t} \qquad g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_o \cong \frac{I_{CQ}}{V_{AF}}$$

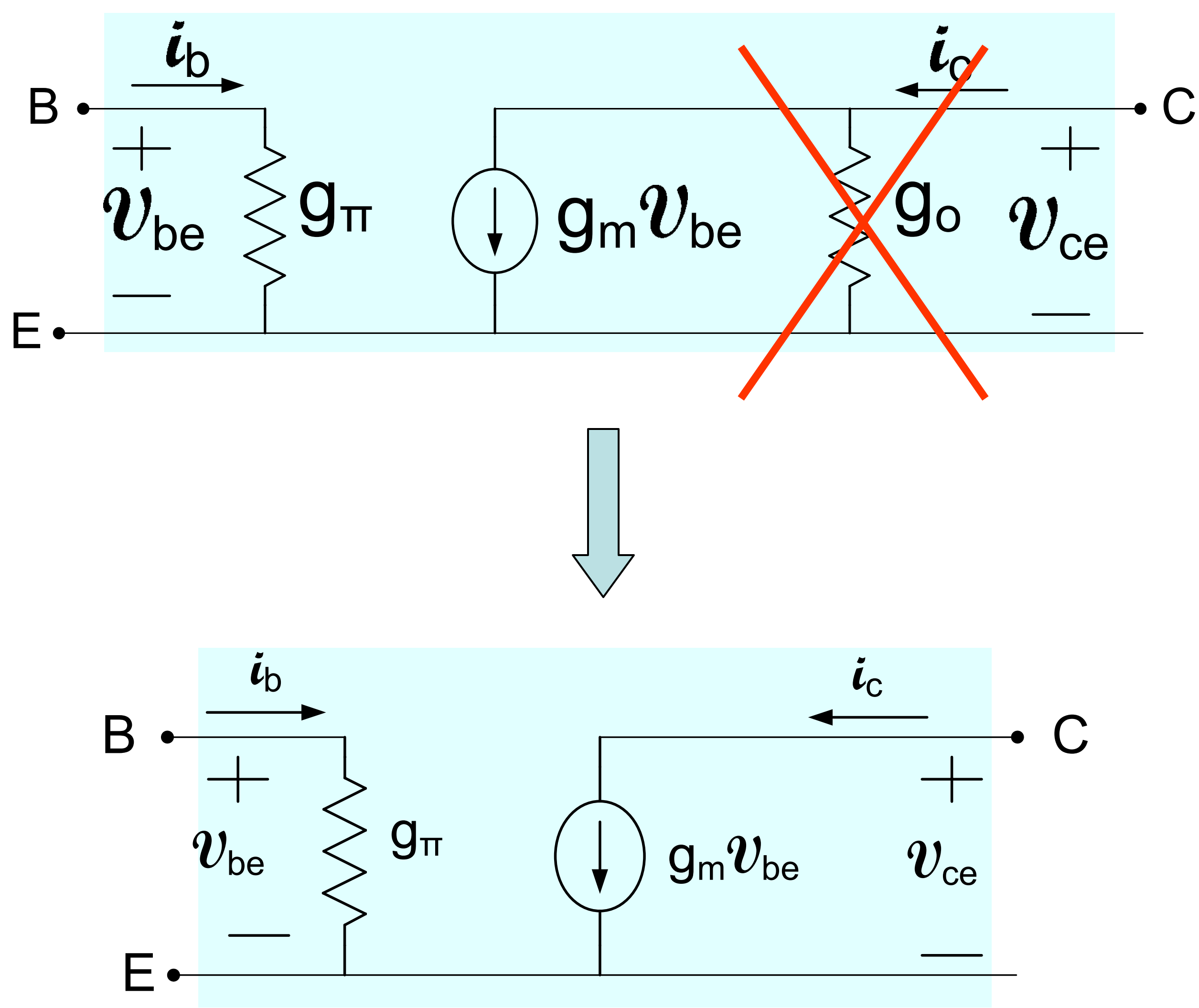
$$\frac{g_m}{g_\pi} = \frac{\left[\dfrac{I_Q}{V_t}\right]}{\left[\dfrac{I_Q}{\beta V_t}\right]} = \beta$$

$$\frac{g_\pi}{g_o} = \frac{\left[\dfrac{I_Q}{\beta V_t}\right]}{\left[\dfrac{I_Q}{V_{AF}}\right]} = \frac{V_{AF}}{\beta V_t} \approx \frac{200V}{100 \bullet 26mV} = 77$$

$$g_m >> g_\pi >> g_o$$

Often the go term can be neglected in the small signal model because it is so small

Simplified small signal model



Comparison of BJT and MOSFET

Comparison of MOSFET and BJT

BJT

$$g_m = \frac{I_{CQ}}{V_t}$$

MOSFET

$$g_m = \frac{\mu C_{OX} W}{L} V_{EB}$$

$$g_m = \sqrt{\frac{2\mu C_{OX} W}{L}} \sqrt{I_{DQ}}$$

$$g_m = \frac{2I_{DQ}}{V_{EBQ}}$$

$$\frac{g_{mBJT}}{g_{mMOS}} = \frac{\dfrac{I_{CQ}}{V_t}}{\dfrac{2I_{DQ}}{V_{EB}}}$$

The transconductance of the BJT is typically much larger than that of the MOSFET (and larger is better!)

This is due to the exponential rather than quadratic output/input relationship

Comparison of MOSFET and BJT

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$$g_m = \sqrt{\frac{2\mu C_{OX} W}{L}} \sqrt{I_{DQ}}$$

$$g_m = \frac{2I_{DQ}}{V_{EBQ}}$$

$$\frac{g_{mBJT}}{g_{mMOS}} = \frac{\frac{I_{CQ}}{V_t}}{\frac{2I_{DQ}}{V_{EB}}} = \frac{V_{EB}}{2V_t} = \begin{cases} \frac{V_{EB}}{50\text{mV}} > \frac{100\text{mV}}{50\text{mV}} = 2 & \text{if } V_{EB} = 100\text{mV} \\ \frac{V_{EB}}{50\text{mV}} > \frac{2\text{V}}{50\text{mV}} = 40 & \text{if } V_{EB} = 2\text{V} \end{cases}$$

The transconductance of the BJT is typically much larger than that of the MOSFET (and larger is better)

This is due to the exponential rather than quadratic output/input relationship

Comparison of MOSFET and BJT

BJT

$$g_o \cong \frac{I_{CQ}}{V_{AF}}$$

MOSFET

$$g_o = \lambda I_{DQ}$$

$$\frac{g_{oBJT}}{g_{oMOS}} = \frac{\frac{I_{CQ}}{V_{AF}}}{\lambda I_{DQ}} = \frac{1}{\lambda V_{AF}} \approx \frac{1}{.01V^{-1}200V} = 0.5$$

The output conductances are comparable but that of the BJT is usually modestly smaller (and smaller is better!)

Comparison of MOSFET and BJT

BJT

$$g_\pi = \frac{I_{CQ}}{\beta V_t}$$

MOSFET

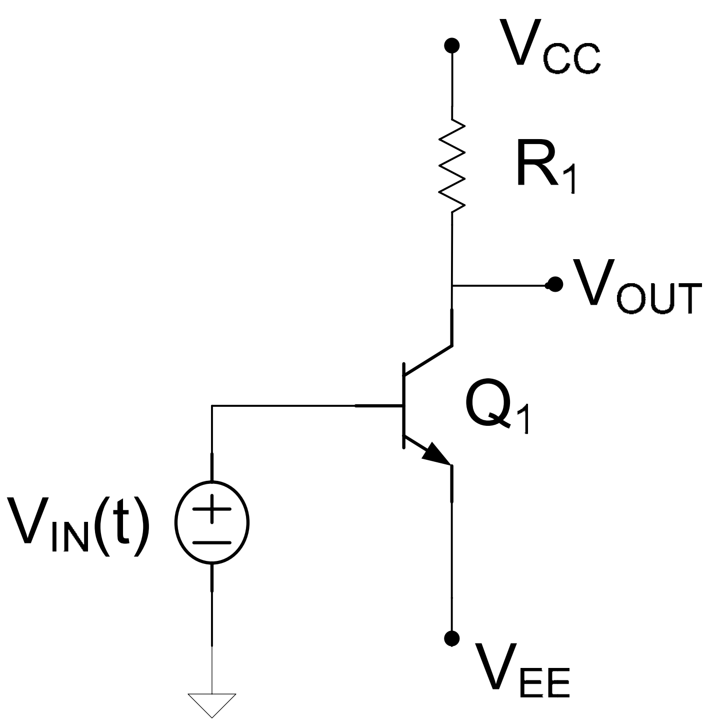
$$g_\pi = 0$$

g_π is the reciprocal of the input impedance

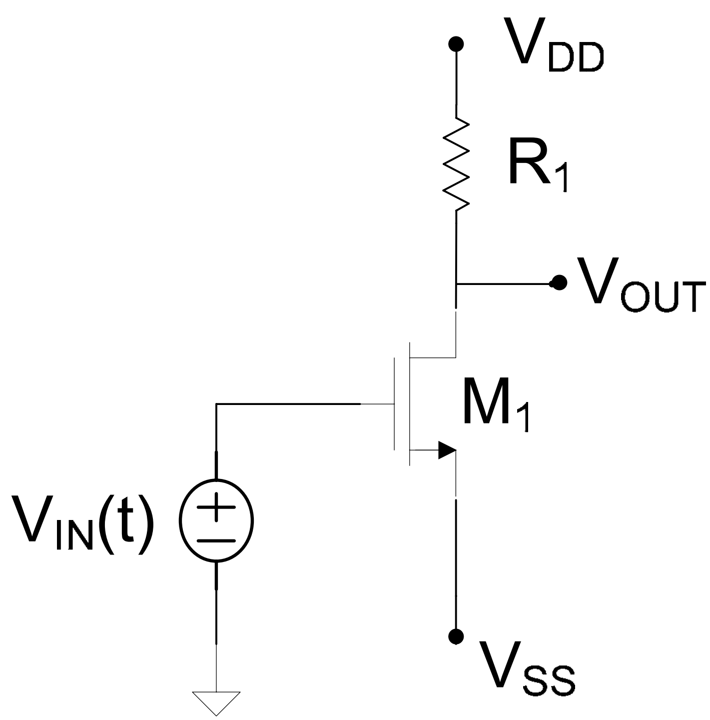
g_π of a MOSFET is much smaller than that of a BJT (and smaller is better!)

Comparison of MOSFET and BJT

BJT



MOSFET



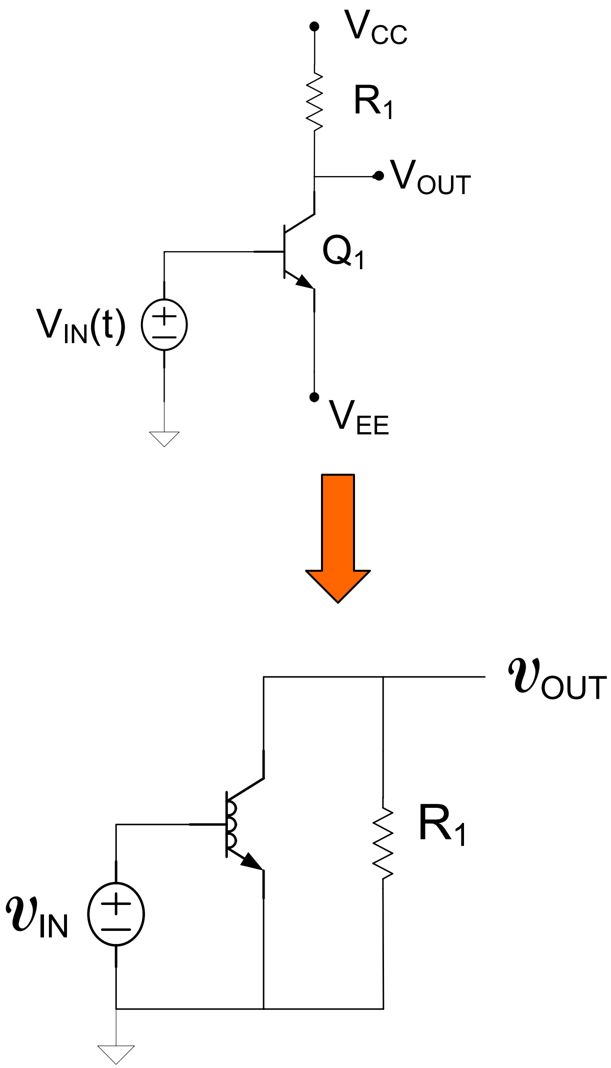
Assume BJT operating in FA region, MOSFET operating in Saturation

Assume same quiescent output voltage and same resistor R_1

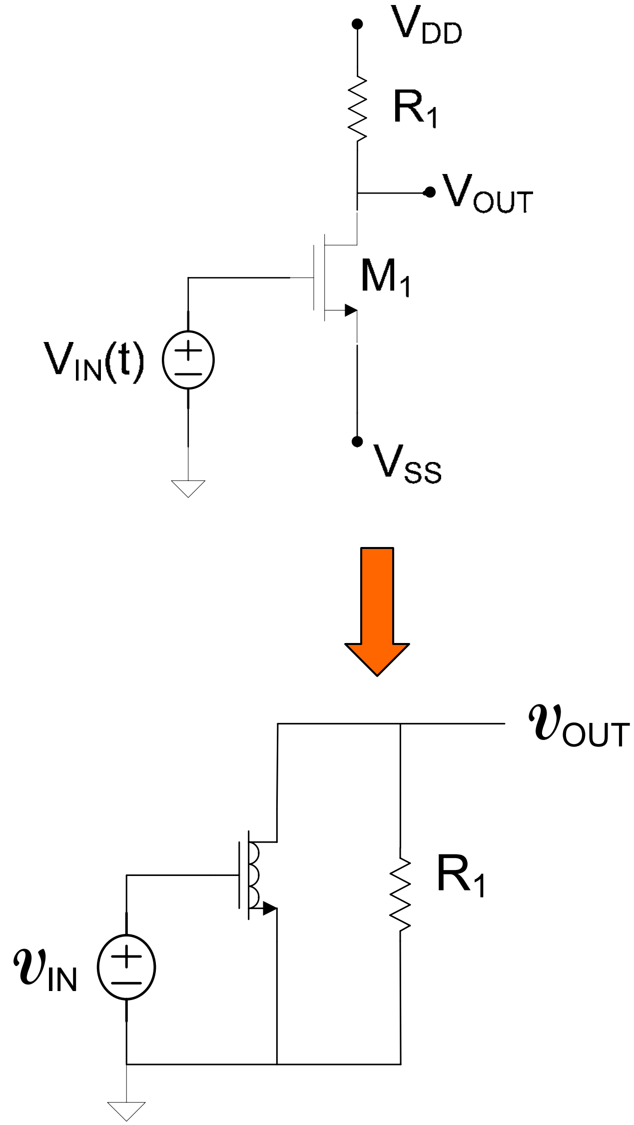
One of the most widely used amplifier architectures

Comparison of MOSFET and BJT

BJT

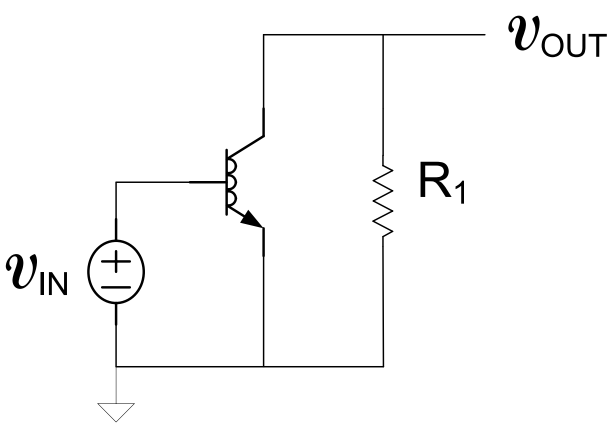


MOSFET

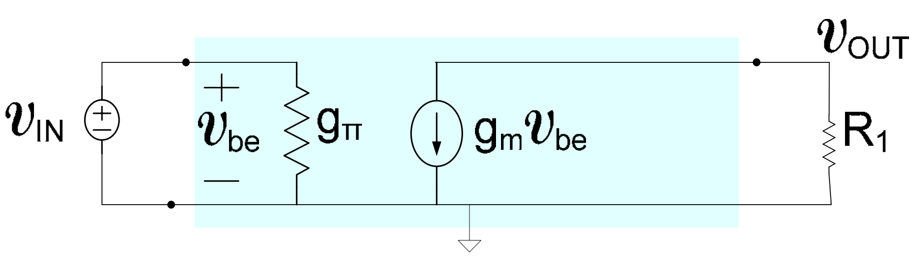


Comparison of MOSFET and BJT

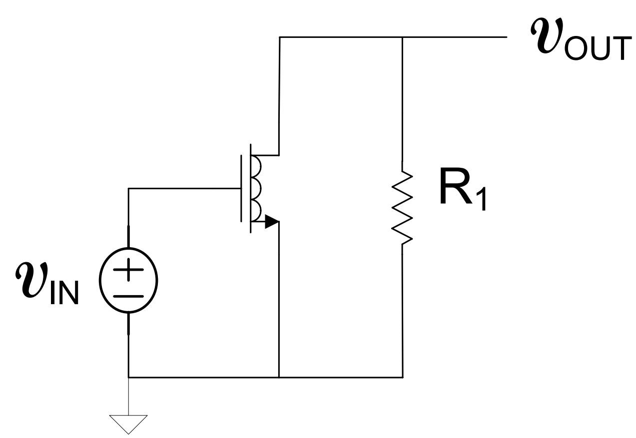
BJT



assume g_o can be neglected



MOSFET



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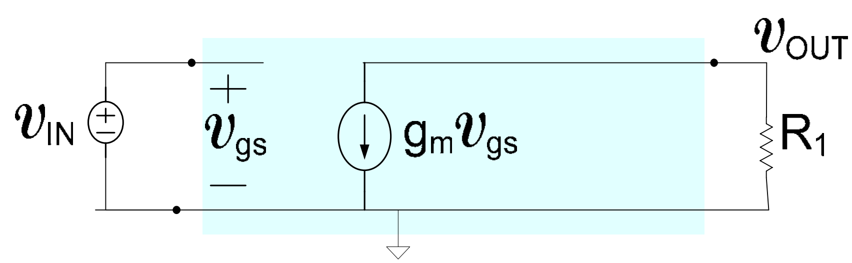
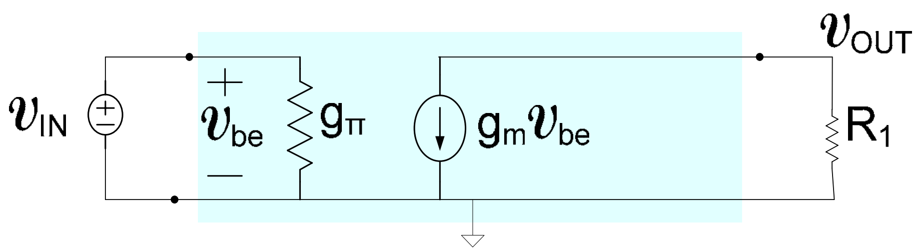


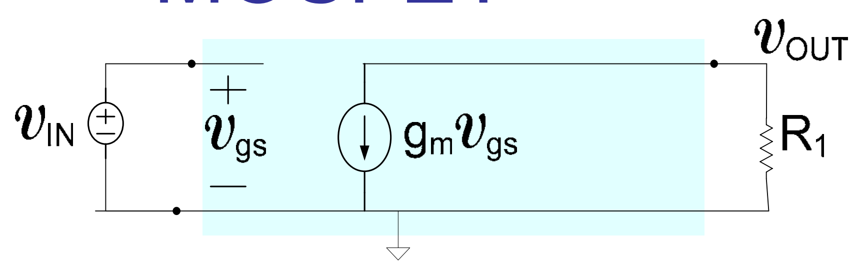
Comparison of MOSFET and BJT

BJT



$$v_{OUT} = -g_m v_{be} R_1$$
$$v_{IN} = v_{be}$$

MOSFET



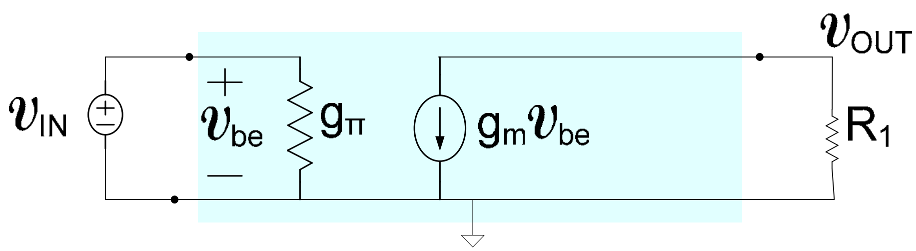
$$v_{OUT} = -g_m v_{gs} R_1$$
$$v_{IN} = v_{gs}$$

$$A_V = \frac{v_{OUT}}{v_{IN}} = -g_m R_1$$

The functional form of the gain is the same for both circuits !

Comparison of MOSFET and BJT

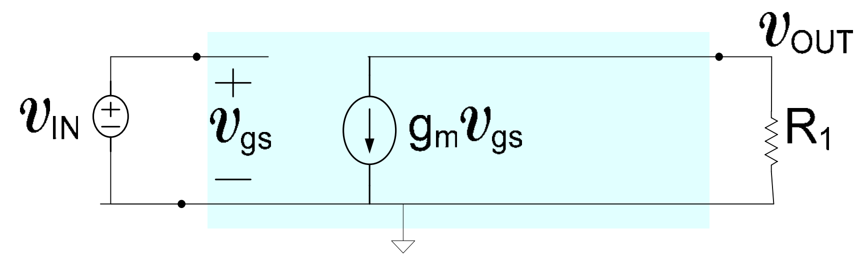
BJT



$$A_V = \frac{v_{OUT}}{v_{IN}} = -g_m R_1$$

$$A_{VBJT} = -\frac{I_{CQ} R_1}{V_t}$$

MOSFET

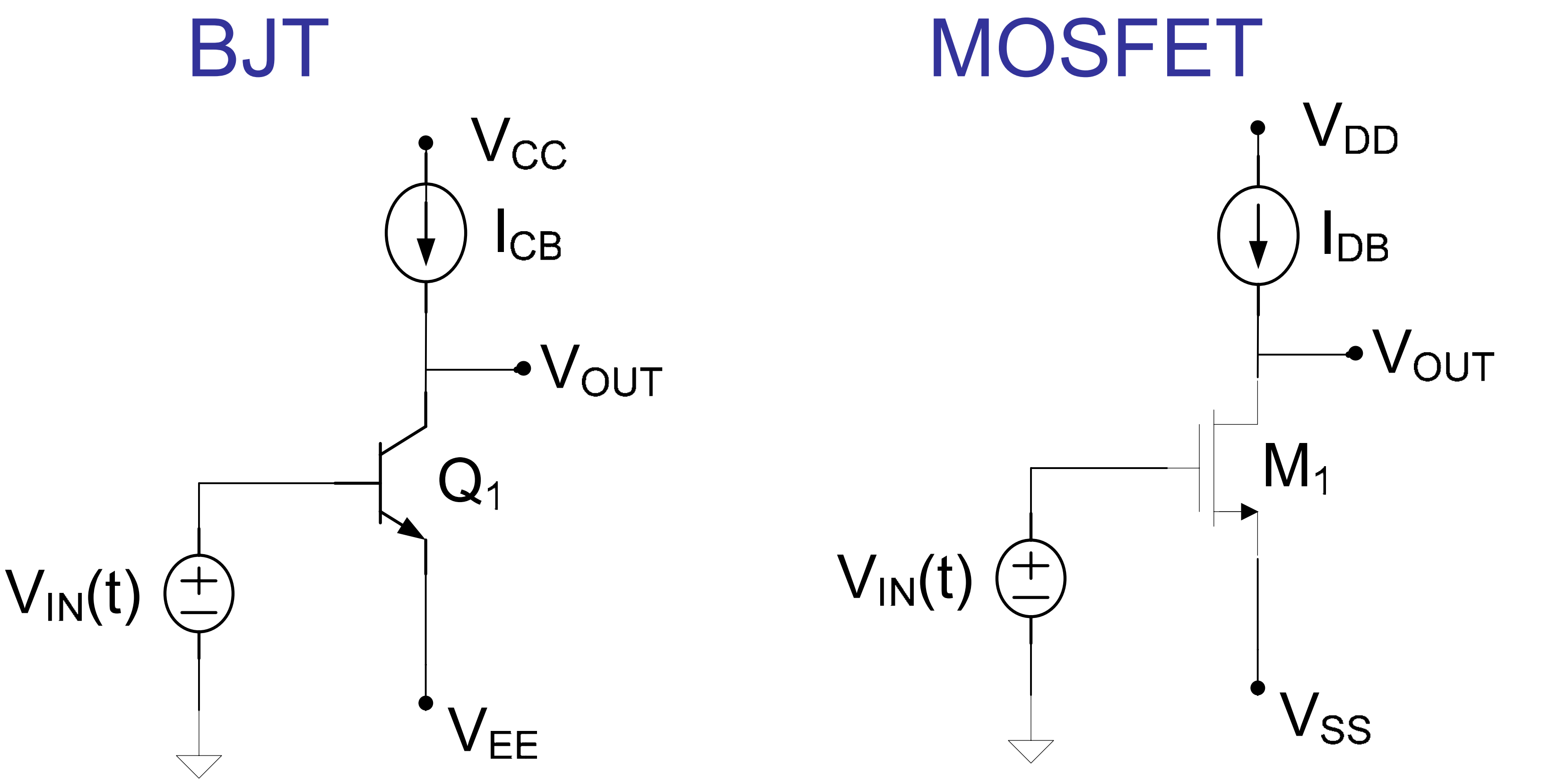


$$A_V = \frac{v_{OUT}}{v_{IN}} = -g_m R_1$$

$$A_{VMOS} = -\frac{2I_{DQ} R_1}{V_{EB}}$$

For the same power level and the same quiescent voltage drop across R_1, the BJT will generally have a much larger gain since usually $V_t \ll V_{EB}$

Comparison of MOSFET and BJT



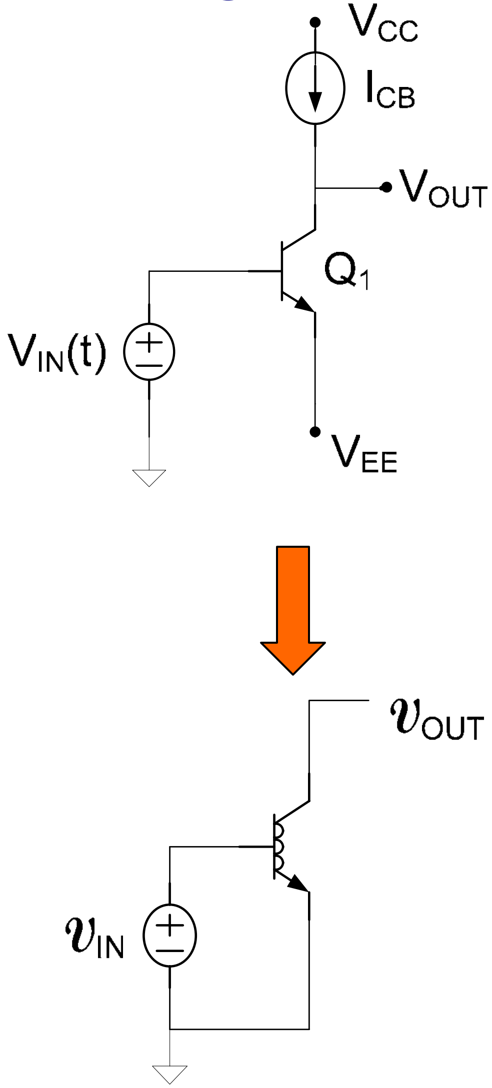
Assume BJT operating in FA region, MOSFET operating in Saturation
Assume same bias current

One of the most widely used amplifier architectures in integrated applications

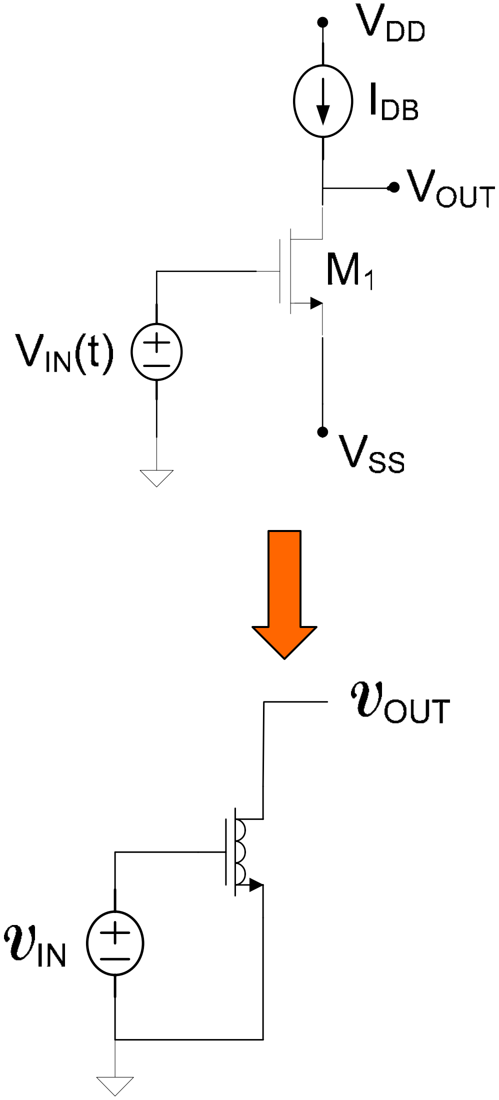
Special Case of Previous Architecture

Comparison of MOSFET and BJT

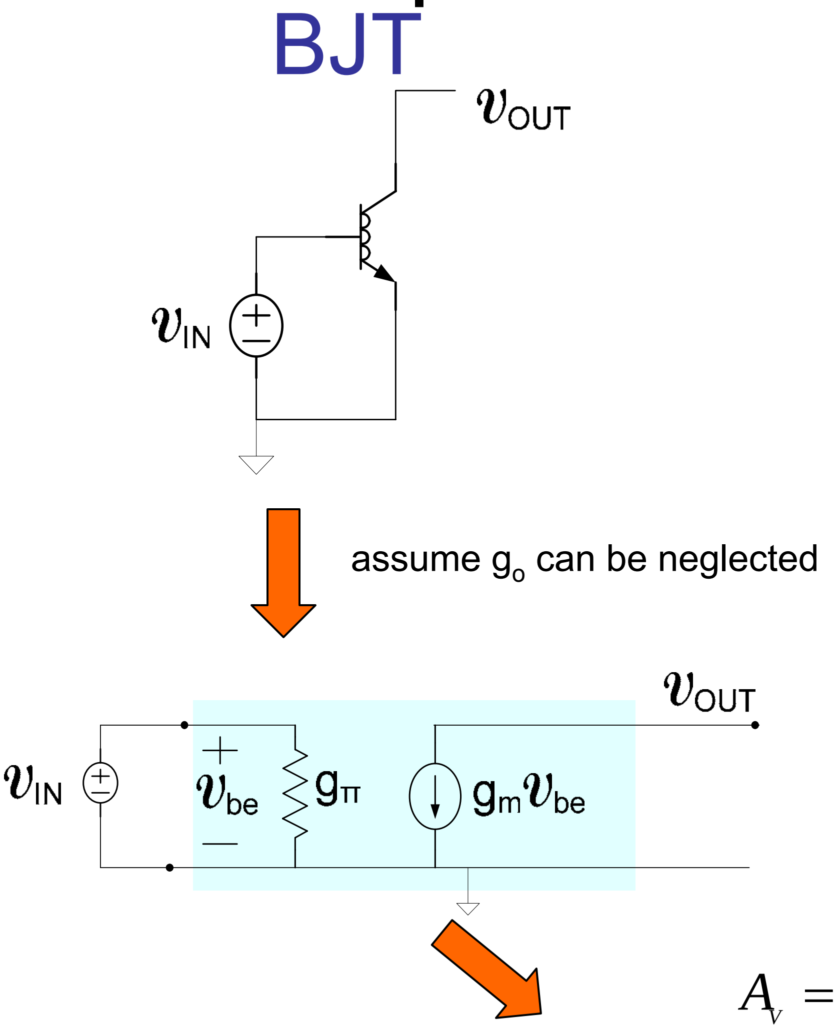
BJT

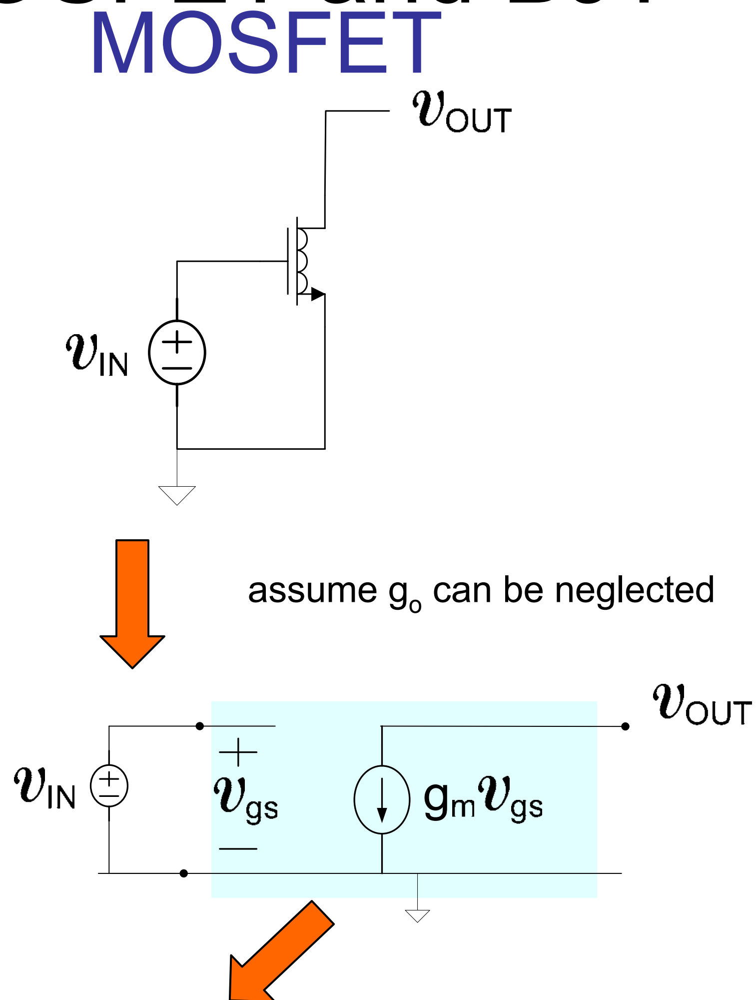


MOSFET



Comparison of MOSFET and BJT



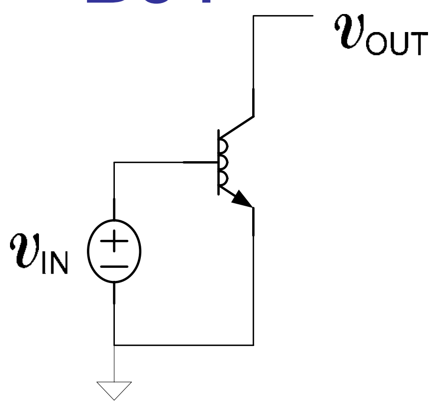


$$A_V = \frac{v_{OUT}}{v_{IN}} = -\infty$$

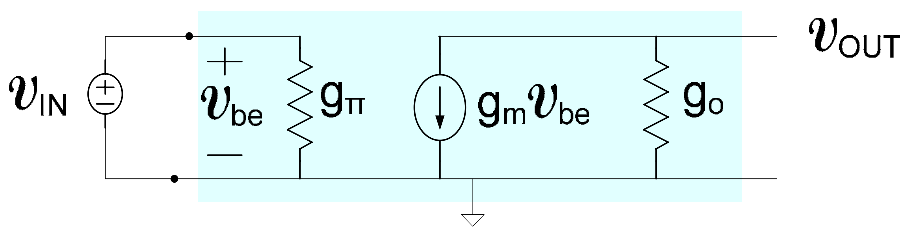
- A_V is unrealistically large
- Must include more accurate small-signal model !

Comparison of MOSFET and BJT

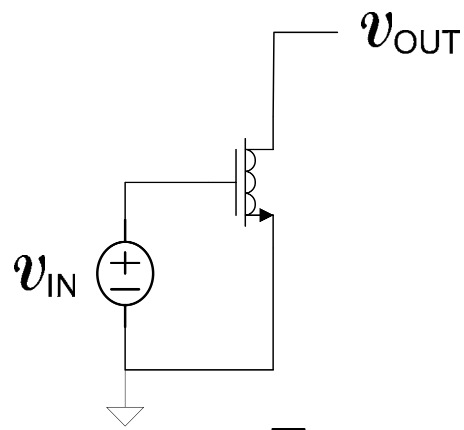
BJT



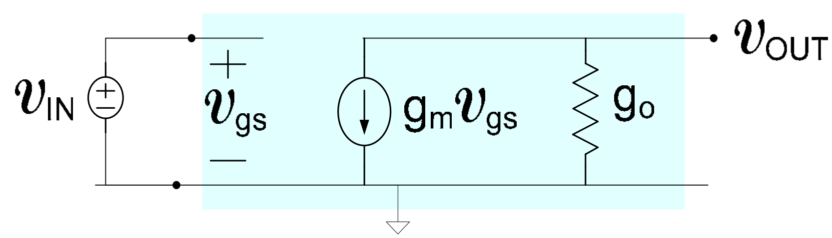
include g_o effects



MOSFET



include g_o effects



$$A_V = \frac{v_{OUT}}{v_{IN}} = -\frac{g_m}{g_o}$$

Functional form of gain is the same for both circuits

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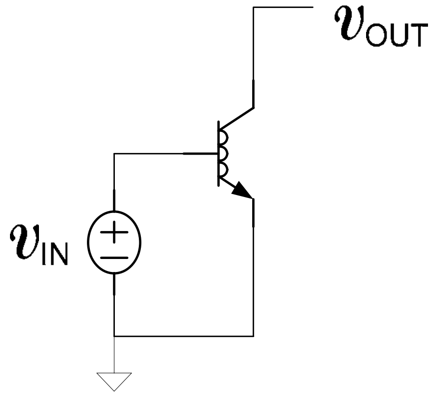
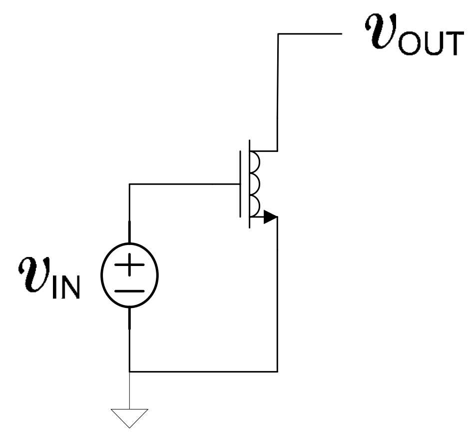
BJT



$$A_V = \frac{v_{OUT}}{v_{IN}} = -\frac{g_m}{g_o}$$

$$A_{VBJT} = -\frac{I_{CQ}/V_t}{I_{CQ}/V_{AF}} = -\frac{V_{AF}}{V_t}$$

MOSFET



$$A_V = \frac{v_{OUT}}{v_{IN}} = -\frac{g_m}{g_o}$$

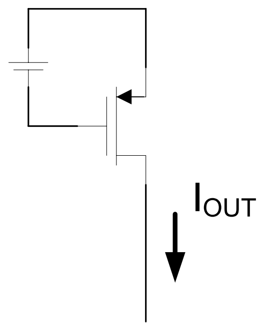
$$A_{VMOS} = -\frac{2I_{DQ}/V_{EB}}{\lambda I_{DQ}} = -\frac{2}{\lambda V_{EB}}$$

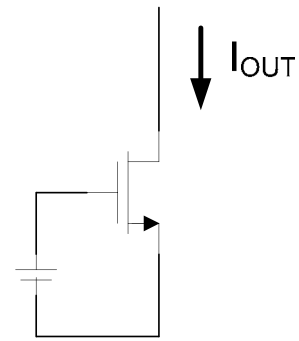
- BJT Gain is Very Large and Independent of Operating Point
- MOS Gain is dependent upon operating conditions (V_{EB})
- V_{AF} and 2/λ are comparable for large MOS devices, V_{AF} considerably larger than 2/λ for short devices
- Practically, $V_t \ll V_{EB}$
- BJT gain typically much larger than MOS gain for this configuration too

Student Question

Can a single transistor be used to realize the current source?

Yes – it provides reasonable performance but there are some limitations





Current sources often characterized by their nominal output current, their small signal output impedance, and their output signal swing

Nominal output current:

$$I_{OUT} \cong \mu C_{OX} \frac{W}{2L} \left(V_{EB}\right)^2$$

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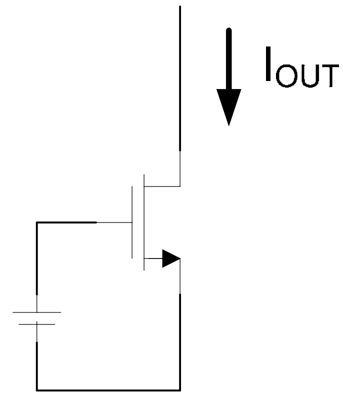
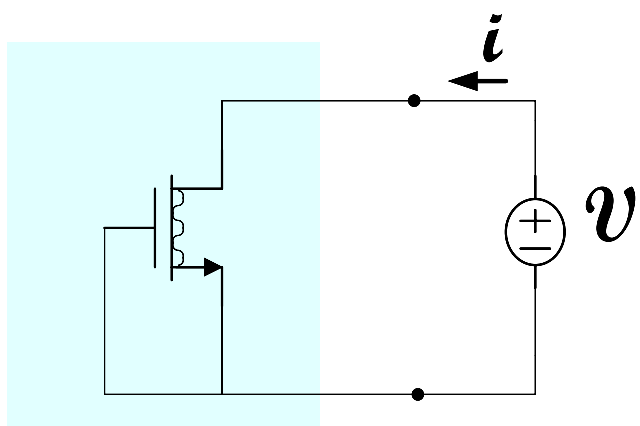
Can a single transistor be used to realize the current source?

Output impedance:





$$R_{out} = \frac{v}{i}$$

Student Question

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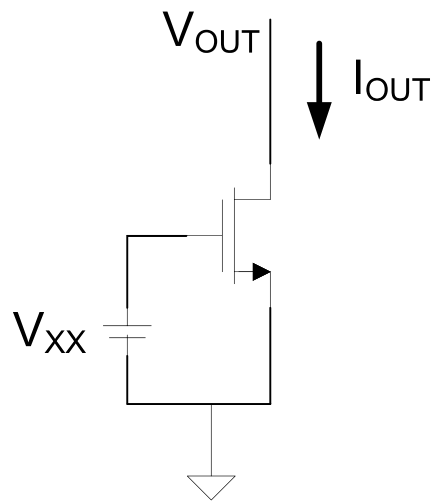
Output signal swing:



To maintain saturation region operation

$V_{DS}>V_{GS}-V_T$

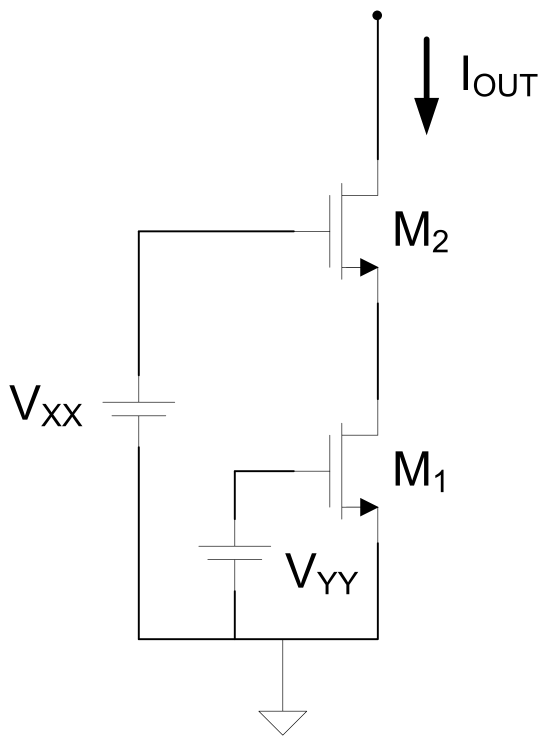
$V_{OUT}>V_{XX}-V_T$

Student Question

Are there better current source circuits?

Yes – and most focus on improving either the signal swing or the output impedance

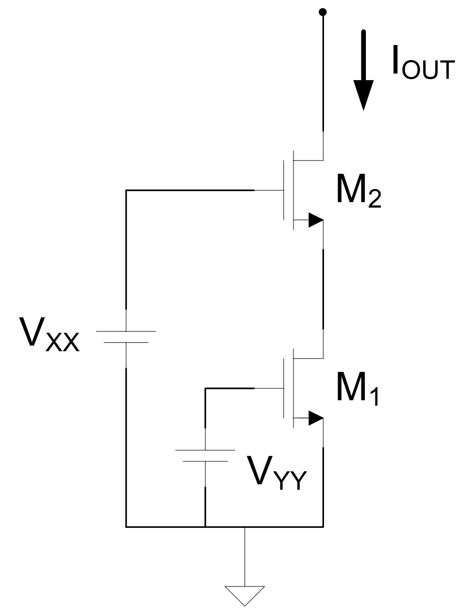
High output impedance current source:



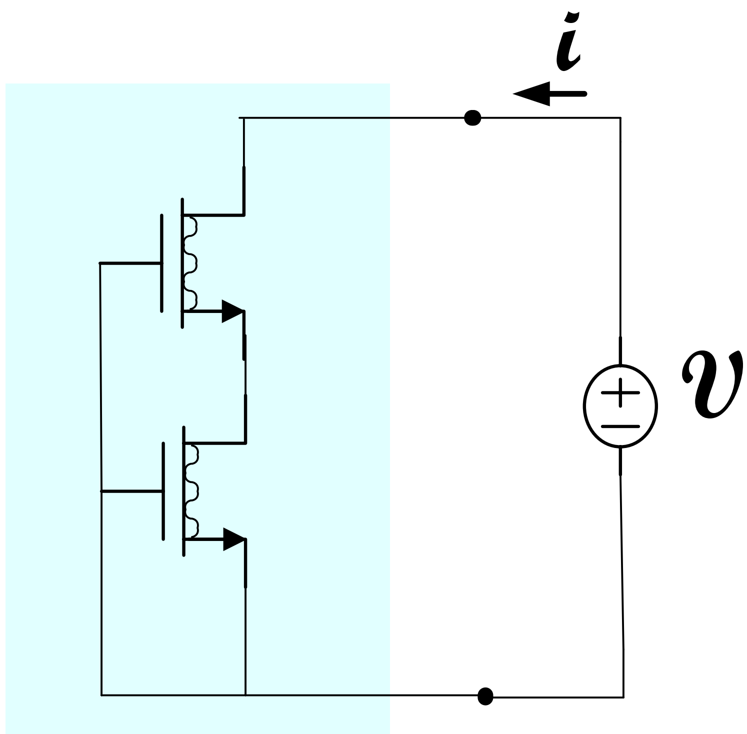
$$I_{OUT} \cong \mu C_{OX} \frac{W_1}{2L_1} \left(V_{EB1}\right)^2$$

Student Question

Are there better current source circuits?



Output Impedance:



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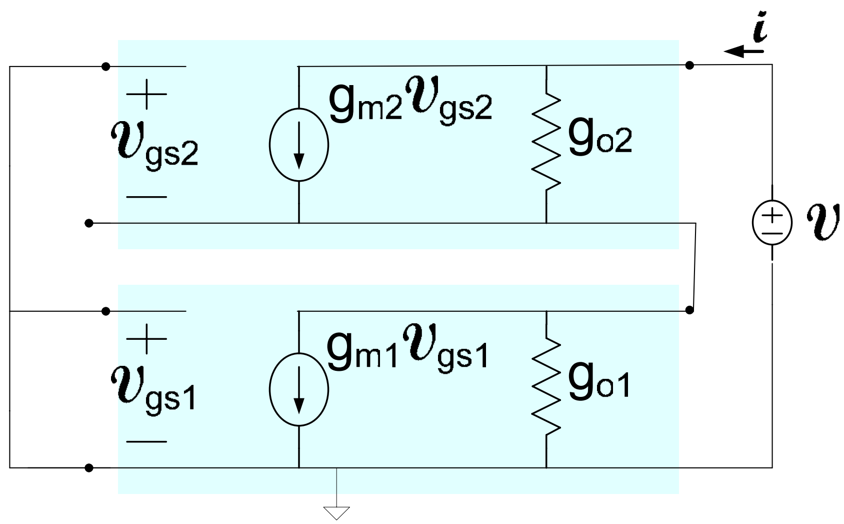
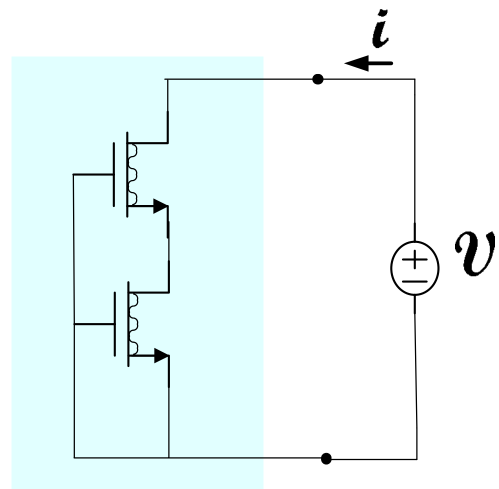
Output Impedance:





$$R_{out} = \frac{v}{i}$$

$$\left.\begin{aligned} i &= (v - v_1)g_{o2} + g_{m2}v_{gs2} \\ v_1(g_{o1} + g_{o2}) &= g_{m2}v_{gs2} + g_{o2}v \\ v_1 &= -v_{gs2} \end{aligned}\right\} \quad R_{out} = \frac{v}{i} = \frac{g_{m2} + g_{o1} + g_{o2}}{g_{o1} + g_{o2}} \cong \left[\frac{1}{g_{o1}}\right]\frac{g_{m2}}{g_{o2}} >> \left[\frac{1}{g_{o1}}\right]$$